Topic 6

Circles

Bronze, Silver, Gold and Platinum Worksheets for AS Level Mathematics

Teacher Notes

These Bronze, Silver and Gold worksheets are designed to be used either straight after the content has been taught or as part of a skills gap analysis, especially as students move into year 13.

They are drawn from the latest specification questions and legacy questions. The papers are between 25 and 35 marks.

The topic number on this worksheet relates to the corresponding chapter number in the 'Pearson Edexcel AS and A Level Mathematics: Pure Mathematics Year 1/AS' textbook.

Non-Calculator Questions

The new specification allows calculators to be used in all papers. **We have, however, put these questions together with the intention that students can complete them without a calculator.** It's important for pupils to be able to maintain their non-calculator skills, especially on topics such as surds or indices, to support question that use the keywords "show that" or "prove". If you wish to ease the difficulty slightly then you can, of course, allow students to attempt them with the support of a calculator.

Quick Links

(Press Ctrl, as you click with your mouse to follow these links)

- Bronze Questions
- Bronze Mark Scheme
- Silver Questions
- Silver Mark Scheme
- Gold Questions
- Gold Mark Scheme

The Platinum Questions below are taken from the Advanced Extension Award. You can use these in class as high level problem solving questions, either with individual students or as group problem solving exercises. On the Advanced Extension Award students, typically, need to get around 50% to get a Merit and around 70% to get a distinction.

- Platinum Questions
- Platinum Mark Schemes

Extension and Enrichment

If you have students that have enjoyed the challenge of the Gold questions, then they should have a go at the more challenging question from our Advanced Extension Award (AEA) papers. The Mathematics AEA is a single, 3 hour non-calculator paper, taken at the end of year 13. It helps students to develop high level problem solving and proof skills. It is entirely based on the content of the A Level Mathematics Course. No extra material needs to be covered to take the AEA in Mathematics. A second important difference is that marks are awarded for the clarity and quality of their solution. Developing this key skill, alongside the extra problem-solving experience, can pay dividends in the way they approach A Level Mathematics and Further Mathematics problems.

More information about the Advanced Extension Award can be found here on the Pearson Edexcel Website, or here on the Maths Emporium

Bronze Questions

Calculators may not be used

The total mark for this section is 29

Q1

A circle C has centre $(-1, 7)$ and passes through the point $(0, 0)$. Find an equation for C

(Total for Question 1 is 4 marks)

Q2

The circle C has equation $x^2 + y^2 + 4x - 2y - 11 = 0$

Find

(a) the coordinates of the centre of C,

(2)

(b) the radius of C,

(2)

(c) the coordinates of the points where C crosses the y-axis,
giving your answers as simplified surds.

(4)

(Total for Question 2 is 8 marks)

Q3

The points A and B have coordinates $(-2, 11)$ and $(8, 1)$ respectively.

Given that AB is a diameter of the circle C,

(a) show that the centre of C has coordinates $(3, 6)$,

(1)

(b) find an equation for C.

(4)

(c) Verify that the point $(10, 7)$ lies on C.

(1)

(d) Find an equation of the tangent to C at the point $(10, 7)$, giving your answer in the form $y = mx + c$, where m and c are constants.

(4)

(Total for Question 3 is 10 marks)

Q4

The circle C, with centre A, passes through the point P with coordinates $(-9, 8)$ and the point Q with coordinates $(15, -10)$.

Given that PQ is a diameter of the circle C,

(a) find the coordinates of A

(2)

(b) find an equation for C

(3)

A point R also lies on the circle C.
Given that the length of the chord PR is 20 units,

(c) find the length of the shortest distance from A to the chord PR.
Give your answer as a surd in its simplest form.

(2)

(Total for Question 4 is 7 marks)

End of Questions

Bronze Mark Scheme

Q1

Question number	Scheme	Marks
	The equation of the circle is $(x+1)^2+(y-7)^2=(r^2)$	M1 A1
	The radius of the circle is $\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $(x+1)^2+(y-7)^2=50$ or equivalent	A1 (4)
		4
Notes	**M1** is for this expression on left hand side– allow *errors in sign* of 1 and 7. **A1** correct signs (just LHS) **M1** is for Pythagoras or substitution into equation of circle to give r or r^2 Giving this value as diameter is **M0** **A1**, cao for cartesian equation with numerical values but allow $(\sqrt{50})^2$ or $(5\sqrt{2})^2$ or any exact equivalent A correct answer implies a correct method – so answer given with no working earns all four marks for this question.	
Alternative method	Equation of circle is $x^2+y^2\pm2x\pm14y+c=0$	M1
	Equation of circle is $x^2+y^2+2x-14y+c=0$	A1
	Uses (0,0) to give $c=0$, **or** finds $r=\sqrt{(-1)^2+7^2}=\sqrt{50}$ or $5\sqrt{2}$ or $r^2=50$	M1
	So $x^2+y^2+2x-14y=0$ or equivalent	A1

Q2

Question Number	Scheme		Marks
(a)	$x^2+y^2+4x-2y-11=0$ $\left\{\underline{(x+2)^2-4}+\underline{(y-1)^2-1}-11=0\right\}$ Centre is $(-2,\,1)$.	 $(\pm2,\pm1)$, see notes. $(-2,\,1)$.	 M1 A1 cao [2]
(b)	$(x+2)^2+(y-1)^2=11+1+4$ So $r=\sqrt{11+1+4}\ \Rightarrow\ r=4$	$r=\sqrt{11\pm"1"\pm"4"}$ 4 or $\sqrt{16}$ (Award A0 for ±4).	M1 A1 [2]
(c)	When $x=0$, $y^2-2y-11=0$ $y=\dfrac{2\pm\sqrt{(-2)^2-4(1)(-11)}}{2(1)}\ \left\{=\dfrac{2\pm\sqrt{48}}{2}\right\}$ So, $y=1\pm2\sqrt{3}$	Putting $x=0$ in C or their C. $y^2-2y-11=0$ or $(y-1)^2=12$, etc Attempt to use formula or a method of completing the square in order to find $y=\dots$ $1\pm2\sqrt{3}$	M1 A1 aef M1 A1 cao cso [4] 8

	Note: Please mark parts (a) and (b) together. Answers only in (a) and/or (b) get full marks. **Note in part (a) the marks are now M1A1 and not B1B1 as on ePEN.**
(a)	M1: for $(\pm2,\pm1)$. Otherwise, M1 for an attempt to complete the square eg. $\underline{(x\pm2)^2\pm\alpha}$, $\alpha\neq0$ or $\underline{(y\pm1)^2\pm\beta}$, $\beta\neq0$. M1A1: Correct answer of $(-2,1)$ stated from any working gets M1A1.
(b)	M1: to find the radius using 11, "1" and "4", ie. $r=\sqrt{11\pm"1"\pm"4"}$. By applying this method candidates will usually achieve $\sqrt{16}$, $\sqrt{6}$, $\sqrt{8}$ or $\sqrt{14}$ and not 16, 6, 8 or 14. <u>Note:</u> $(x+2)^2+(y-1)^2=-11-5=-16\Rightarrow r=\sqrt{16}=4$ should be awarded M0A0. <u>Alternative:</u> M1 in part (a): For comparing with $x^2+y^2+2gx+2fy+c=0$ to write down centre $(-g,-f)$ directly. Condone sign errors for this M mark. M1 in part (b): For using $r=\sqrt{g^2+f^2-c}$. Condone sign errors for this method mark. $(x+2)^2+(y-1)^2=16\Rightarrow r=8$ scores M0A0, but $r=\sqrt{16}=8$ scores M1A1 isw.
(c)	1st M1: Putting $x=0$ in either $x^2+y^2+4x-2y-11=0$ or their circle equation usually given in part (a) or part (b). 1st A1 for a correct equation in y **in any form** which can be implied by later working. 2nd M1: See rules for using the formula. Or completing the square on a 3TQ to give $y=a\pm\sqrt{b}$, where $\sqrt{b}$ is a surd, $b\neq$ their 11 and $b>0$. This mark should not be given for an attempt to factorise. 2nd A1: Need exact pair in simplified surd form of $\{y=\}\,1\pm2\sqrt{3}$. This mark is also cso. Do not need to see $(0,1+2\sqrt{3})$ and $(0,1-2\sqrt{3})$. Allow 2nd A1 for bod $(1+2\sqrt{3},0)$ and $(1-2\sqrt{3},0)$. Any incorrect working in (c) gets penalised the final accuracy mark. So, **<u>beware:</u>** incorrect $(x-2)^2+(y-1)^2=16$ leading to $y^2-2y-11=0$ and then $y=1\pm2\sqrt{3}$ scores M1A1M1A0. **<u>Special Case for setting $y=0$:</u>** Award SC: M0A0M1A0 for an attempt at applying the formula $x=\dfrac{-4\pm\sqrt{(-4)^2-4(1)(-11)}}{2(1)}\ \left\{=\dfrac{-4\pm\sqrt{60}}{2}=-2\pm\sqrt{15}\right\}$ Award SC: M0A0M1A0 for completing the square to their equation in x which will usually be $x^2+4x-11=0$ to give $a\pm\sqrt{b}$, where $\sqrt{b}$ is a surd, $b\neq$ their 11 and $b>0$. <u>Special Case</u>: For a candidate not using $\pm$ but achieving one of the correct answers then award SC: M1A1M1A0 for one of either $y=1+2\sqrt{3}$ or $y=1-2\sqrt{3}$ or $y=1+\sqrt{12}$ or $y=1-\sqrt{12}$.

Q3

Question Number	Scheme		Marks
(a)	$C\left(\frac{-2+8}{2}, \frac{11+1}{2}\right) = C(3, 6)$ **AG**	Correct method (no errors) for finding the mid-point of AB giving $(3, 6)$	B1*
			(1)
(b)	$(8-3)^2 + (1-6)^2$ or $\sqrt{(8-3)^2 + (1-6)^2}$ or	Applies distance formula in order to find the radius.	M1
	$(-2-3)^2 + (11-6)^2$ or $\sqrt{(-2-3)^2 + (11-6)^2}$	Correct application of formula.	A1
	$(x-3)^2 + (y-6)^2 = 50$ $\left(\text{or } \left(\sqrt{50}\right)^2 \text{ or } \left(5\sqrt{2}\right)^2\right)$	$(x \pm 3)^2 + (y \pm 6)^2 = k$, k is a positive value.	M1
		$(x-3)^2 + (y-6)^2 = 50$ (Not 7.07^2)	A1
			(4)
(c)	{For $(10, 7)$,} $\underline{(10-3)^2 + (7-6)^2 = 50}$, {so the point lies on C.}		<u>B1</u>
			(1)
(d)	{Gradient of radius} $= \frac{7-6}{10-3}$ or $\frac{1}{7}$	This must be seen in part (d).	B1
	Gradient of tangent $= \frac{-7}{1}$	Using a perpendicular gradient method.	M1
	$y - 7 = -7(x - 10)$	$y - 7 = (\text{their gradient})(x - 10)$	M1
	$y = -7x + 77$	$y = -7x + 77$ or $y = 77 - 7x$	A1 cao
			(4)
			[10]
	<u>Notes</u>		
(a)	Alternative method: $C\left(-2 + \frac{8--2}{2}, 11 + \frac{1-11}{2}\right)$ or $C\left(8 + \frac{-2-8}{2}, 1 + \frac{11-1}{2}\right)$		
(b)	You need to be convinced that the candidate is attempting to work out the radius and not the diameter of the circle to award the first M1. Therefore allow 1st M1 generously for $\frac{(-2-8)^2 + (11-1)^2}{2}$ Award 1st M1A1 for $\frac{(-2-8)^2 + (11-1)^2}{4}$ or $\frac{\sqrt{(-2-8)^2 + (11-1)^2}}{2}$. Correct answer in (b) with no working scores full marks.		
(c)	B1 awarded for correct verification of $\underline{(10-3)^2 + (7-6)^2 = 50}$ with no errors. Also to gain this mark candidates need to have the correct equation of the circle either from part (b) or re-attempted in part (c). They cannot verify $(10, 7)$ lies on C without a correct C. Also a candidate could either substitute $x = 10$ in C to find $y = 7$ or substitute $y = 7$ in C to find $x = 10$.		

Question Number	Scheme	Marks
(d)	2nd M1 mark also for the complete method of applying $7 = (\text{their gradient})(10) + c$, finding c. **Note**: Award 2nd M0 in (d) if their numerical gradient is either 0 or ∞. **<u>Alternative:</u>** For first two marks (differentiation): $2(x-3) + 2(y-6)\frac{dy}{dx} = 0$ (or equivalent) scores B1. 1st M1 for substituting **both** $x = 10$ and $y = 7$ to find a value for $\frac{dy}{dx}$, which must contain both x and y. (This M mark can be awarded generously, even if the attempted "differentiation" is not "implicit".) **<u>Alternative:</u>** $(10-3)(x-3) + (7-6)(y-6) = 50$ scores B1M1M1 which leads to $y = -7x + 77$.	

Q4

Question Number	Scheme		Marks
(a)	$A\left(\frac{-9+15}{2}, \frac{8-10}{2}\right) = A(3, -1)$	M1: A correct attempt to find the midpoint between P and Q. Can be implied by one of x or y-coordinates correctly evaluated.	M1A1
		A1: $(3, -1)$	
			[2]
(b)	$(-9-3)^2 + (8+1)^2$ or $\sqrt{(-9-3)^2 + (8+1)^2}$ or $(15-3)^2 + (-10+1)^2$ or $\sqrt{(15-3)^2 + (-10+1)^2}$ Uses Pythagoras correctly in order to find the **radius**. Must clearly be identified as the **radius** and may be implied by their circle equation. Or $(15+9)^2 + (-10-8)^2$ or $\sqrt{(15+9)^2 + (-10-8)^2}$ Uses Pythagoras correctly in order to find the **diameter**. Must clearly be identified as the **diameter** and may be implied by their circle equation. This mark can be implied by just 30 clearly seen as the **diameter** or 15 clearly seen as the **radius** (may be seen or implied in their circle equation) **Allow this mark if there is a correct statement involving the radius or the diameter but <u>must be seen in (b)</u>**		M1
	$(x-3)^2 + (y+1)^2 = 225 \ \left(\text{or } (15)^2\right)$	$(x \pm \alpha)^2 + (y \pm \beta)^2 = k^2$ where $A(\alpha, \beta)$ and k is their radius.	M1
	$(x-3)^2 + (y+1)^2 = 225$	Allow $(x-3)^2 + (y+1)^2 = 15^2$	A1
	Accept correct answer only		
			[3]
	Alternative using $x^2 + 2ax + y^2 + 2by + c = 0$		
	Uses $A(\pm\alpha, \pm\beta)$ and $x^2 + 2ax + y^2 + 2by + c = 0$ e.g. $x^2 + 2(-3)x + y^2 + 2(1)y + c = 0$		M1
	Uses P or Q and $x^2 + 2ax + y^2 + 2by + c = 0$ e.g. $(-9)^2 + 2(-3)(-9) + (8)^2 + 2(1)(8) + c = 0 \Rightarrow c = -215$		M1
	$x^2 - 6x + y^2 + 2y - 215 = 0$		A1
(c)	Distance $= \sqrt{15^2 - 10^2}$	$= \sqrt{(\text{their } r)^2 - 10^2}$ or a correct method for the distance e.g. their $r \times \cos\left[\sin^{-1}\left(\frac{10}{\text{their } r}\right)\right]$	M1
	$\left\{= \sqrt{125}\right\} = 5\sqrt{5}$	$5\sqrt{5}$	A1
			[2]

Silver Questions

Calculators may not be used

The total mark for this section is 33

Q1

The circle C has equation

$$x^2 + y^2 - 20x - 24y + 195 = 0$$

The centre of C is at the point M

(a) Find

(i) the coordinates of the point M

(ii) the radius of the circle C

(5)

N is the point with coordinates (25, 32)

(b) Find the length of the line MN

(2)

The tangent to C at a point P on the circle passes through point N

(c) Find the length of the line NP

(2)

(Total for Question 1 is 9 marks)

Q2

The circle C has centre (3, 1) and passes through the point P(8, 3).

(a) Find an equation for C.

(4)

(b) Find an equation for the tangent to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(5)

(Total for Question 2 is 9 marks)

Q3

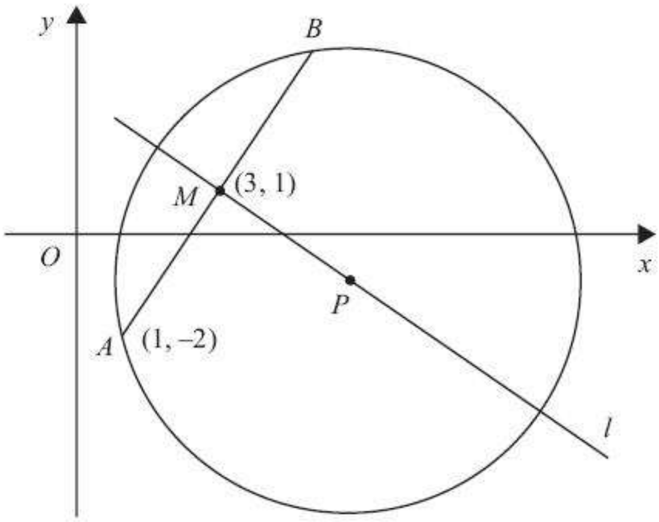


Figure 3

The points A and B lie on a circle with centre P, as shown in Figure 3.
The point A has coordinates $(1, -2)$ and the mid-point M of AB has coordinates $(3, 1)$.
The line l passes through the points M and P.

(a) Find an equation for l.

(4)

Given that the x-coordinate of P is 6,

(b) use your answer to part (a) to show that the y-coordinate of P is -1,

(1)

(c) find an equation for the circle.

(4)

(Total for Question 3 is 9 marks)

Q4

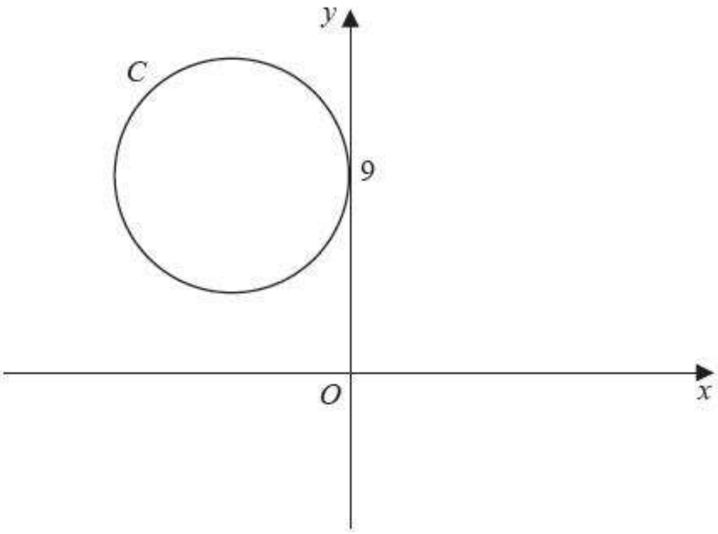


Figure 4

The circle C has radius 5 and touches the y-axis at the point (0, 9), as shown in Figure 4.

(a) Write down an equation for the circle C, that is shown in Figure 4.

(3)

A line through the point $P(8, -7)$ is a tangent to the circle C at the point T

(b) Find the length of PT

(3)

(Total for Question 4 is 6 marks)

End of Questions

Silver Mark Scheme

Q1

Question Number	Scheme		Marks
(a)			
(i)	The centre is at (10, 12)	B1: $x = 10$ B1: $y = 12$	B1 B1
(ii)	Uses $(x-10)^2 + (y-12)^2 = -195 + 100 + 144 \Rightarrow r = ...$		M1
	Completes the square for both x and y in an attempt to find r. $(x \pm "10")^2 \pm a$ and $(y \pm "12")^2 \pm b$ and $+195 = 0, (a, b \neq 0)$ Allow errors in obtaining their r^2 but must find square root		
	$r = \sqrt{10^2 + 12^2 - 195}$	A correct numerical expression for r including the square root and can implied by a correct value for r	A1
	$r = 7$	Not $r = \pm 7$ unless -7 is rejected	A1
			(5)
(a) Way 2	Compares the given equation with $x^2 + y^2 + 2gx + 2fy + c = 0$ to write down centre $(-g, -f)$ i.e. (10, 12)	B1: $x = 10$ B1: $y = 12$	B1B1
	Uses $r = \sqrt{(\pm "10")^2 + (\pm "12")^2 - c}$		M1
	$r = \sqrt{10^2 + 12^2 - 195}$	A correct numerical expression for r	A1
	$r = 7$		A1
			(5)
(b)	$MN = \sqrt{(25 - "10")^2 + (32 - "12")^2}$	Correct use of Pythagoras	M1
	$MN \left(= \sqrt{625}\right) = 25$		A1
			(2)
(c)	$NP = \sqrt{("25"^2 - "7"^2)}$	$NP = \sqrt{(MN^2 - r^2)}$	M1
	$NP \left(= \sqrt{576}\right) = 24$		A1
			(2)
(c) Way 2	$\cos(NMP) = \dfrac{7}{"25"} \Rightarrow NP = "25" \sin(NMP)$	Correct strategy for finding NP	M1
	$NP = 24$		A1
			(2)
			[9]

Q2

Question Number	Scheme	Marks
(a)	$(8-3)^2+(3-1)^2$ or $\sqrt{(8-3)^2+(3-1)^2}$	M1 A1
	$(x\pm3)^2+(y\pm1)^2=k$ or $(x\pm1)^2+(y\pm3)^2=k$ (k a positive value)	M1
	$(x-3)^2+(y-1)^2=29$	A1 (4)
(b)	Gradient of radius $=\dfrac{2}{5}$ (or exact equivalent)	B1
	Gradient of tangent $=\dfrac{-5}{2}$	M1
	$y-3=\dfrac{-5}{2}(x-8)$	M1 A1 ft
	$5x+2y-46=0$ or equivalent	A1 (5)
		(9 marks)

Q3

Question number	Scheme	Marks	
	(a) Gradient of AM: $\dfrac{1-(-2)}{3-1}=\dfrac{3}{2}$ or $\dfrac{-3}{-2}$	B1	
	Gradient of l: $=-\dfrac{2}{3}$ M: use of $m_1m_2=-1$, or equiv.	M1	
	$y-1=-\dfrac{2}{3}(x-3)$ or $\dfrac{y-1}{x-3}=-\dfrac{2}{3}$ $[3y=-2x+9]$ (Any equiv. form)	M1 A1	(4)
	(b) $x=6$: $3y=-12+9=-3$ $y=-1$ (or show that for $y=-1$, $x=6$) (*) (A conclusion is <u>not</u> required).	B1	(1)
	(c) $(r^2=)$ $(6-1)^2+(-1-(-2))^2$ M: Attempt r^2 or r	M1 A1	
	N.B. Simplification is <u>not</u> required to score M1 A1		
	$(x\pm 6)^2+(y\pm 1)^2=k$, $k\neq 0$ (<u>Value</u> for k not needed, could be r^2 or r)	M1	
	$(x-6)^2+(y+1)^2=26$ (or equiv.)	A1	(4)
	Allow $\left(\sqrt{26}\right)^2$ or other exact equivalents for 26. (But… $(x-6)^2+(y--1)^2=26$ scores M1 A0)		
	(Correct answer with no working scores full marks)		
			9
	(a) 2nd M1: eqn. of a straight line through (3, 1) with any gradient except 0 or ∞. <u>Alternative:</u> Using (3, 1) in $y=mx+c$ to find a value of c scores M1, but an equation (general or specific) must be seen. Having coords the <u>wrong way round</u>, e.g. $y-3=-\dfrac{2}{3}(x-1)$, loses the 2nd M mark <u>unless</u> a correct general formula is seen, e.g. $y-y_1=m(x-x_1)$. If the point $P(6,-1)$ is used to find the gradient of MP, maximum marks are (a) B0 M0 M1 A1 (b) B0. (c) 1st M1: Condone <u>one</u> slip, numerical or sign, <u>inside</u> a bracket. Must be attempting to use points $P(6,-1)$ and $A(1,-2)$, or perhaps P and B. (Correct coordinates for B are (5, 4)). 1st M alternative is to use a complete Pythag. method on triangle MAP, n.b. $MP=MA=\sqrt{13}$. <u>Special case:</u> If candidate persists in using <u>their</u> value for the y-coordinate of P instead of the given -1, allow the M marks in part (c) if earned.		

Q4

Question Number	Scheme	Marks
(a)	Equation of form $(x \pm 5)^2 + (y \pm 9)^2 = k$, $k > 0$	M1
	Equation of form $(x-a)^2 + (y-b)^2 = 5^2$, with values for a and b	M1
	$(x+5)^2 + (y-9)^2 = 25 = 5^2$	A1
		(3)
	$P(8, -7)$. Let centre of circle $= X(-5, 9)$	
(b)	$PX^2 = (8 - \text{"}-5\text{"})^2 + (-7 - \text{"}9\text{"})^2$ or $PX = \sqrt{(8--5)^2 + (-7-9)^2}$	M1
	($PX = \sqrt{425}$ or $5\sqrt{17}$) $PT^2 = (PX)^2 - 5^2$ with numerical PX	dM1
	$PT \left\{= \sqrt{400}\right\} = 20$ (allow 20.0)	A1 cso
		(3) [6]
Alternative 2 for (a)	Equation of the form $x^2 + y^2 \pm 10x \pm 18y + c = 0$	M1
	Uses $a^2 + b^2 - 5^2 = c$ with their a and b or substitutes (0, 9) giving $+9^2 \pm 2b \times 9 + c = 0$	M1
	$x^2 + y^2 + 10x - 18y + 81 = 0$	A1 (3)
Alternative 2 for (b)	An attempt to find the point T may result in pages of algebra, but solution needs to reach (-8, 5) or $\left(\frac{-8}{17}, 11\frac{2}{17}\right)$ to get first M1 (even if gradient is found first)	M1
	M1: Use either of the correct points with P (8, -7) and distance between two points formula	dM1
	A1: 20	A1cso (3)
Alternative 3 for (b)	Substitutes (8, -7) into circle **equation** so $PT^2 = 8^2 + (-7)^2 + 10 \times 8 - 18 \times (-7) + 81$	M1
	Square roots to give $PT \left\{= \sqrt{400}\right\} = 20$	dM1A1 (3)
	Notes for Question	
(a)	**The three marks in (a) each require a circle equation – (see special cases which are not circles)** M1: Uses coordinates of centre to obtain LHS of circle equation (RHS must be r^2 or $k > 0$ or a positive value) M1: Uses $r = 5$ to obtain RHS of circle equation as 25 or 5^2 A1: correct circle equation in any equivalent form **Special cases** $(x \pm 5)^2 + (x \pm 9)^2 = (5^2)$ **is not a circle** equation so M0M0A0 Also $(x \pm 5)^2 + (y-9) = (5^2)$ **And** $(x \pm 5)^2 - (y \pm 9)^2 = (5^2)$ **are not circles** and gain M0M0A0 **But** $(x-0)^2 + (y-9)^2 = 5^2$ gains M0M1A0	
(b)	M1: Attempts to find distance from their **centre of circle** to P (or square of this value). If this is called PT and given as answer this is M0. Solution may use letter other than X, as centre was not labelled in the question. N.B. Distance from (0, 9) to (8, -7) is incorrect method and is M0, followed by M0A0. dM1: Applies the **subtraction** form of Pythagoras to find PT or PT^2 (depends on previous method mark for distance from **centre to** P) or uses appropriate complete method involving trigonometry A1: 20 cso	

Pearson

Gold Questions

Calculators may not be used

The total mark for this section is 35

Q1

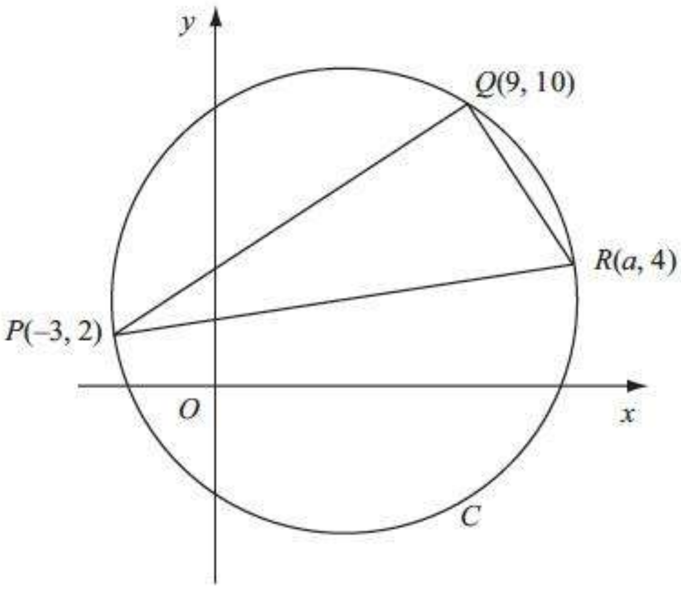


Figure 2

The points P(−3, 2), $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2. Given that PR is a diameter of C,

(a) show that $a = 13$

(3)

(b) find an equation for C

(5)

(2)

(Total for Question 1 is 10 marks)

Q2

The circle C has centre $A(2,1)$ and passes through the point $B(10, 7)$

(a) Find an equation for C

(4)

The line l_1 is the tangent to C at the point B

(b) Find an equation for l_1

(4)

The line l_2 is parallel to l_1 and passes through the mid-point of AB

Given that l_2 intersects C at the points P and Q,

(c) find the length of PQ, giving your answer in its simplest surd form.

(3)

(Total for Question 2 is 11 marks)

Q3

The circle C has equation

$$x^2 + y^2 - 6x + 10y + 9 = 0$$

(a) Find

(i) the coordinates of the centre of C
(ii) the radius of C

(3)

The line with equation $y = kx$, where k is a constant, cuts C at two distinct points.

(b) Find the range of values for k

(6)

(Total for Question 3 is 9 marks)

Q4

A circle C with centre at $(-2, 6)$ passes through the point $(10, 11)$.
The circle C also passes through the point $(10, 1)$ and has the equation
$(x+2)^2 + (y-6)^2 = 13^2$

The tangent to the circle C at the point $(10, 11)$ meets the y axis at the point P
and the tangent to the circle C at the point $(10, 1)$ meets the y axis at the point Q.
Show that the distance PQ is 58 explaining your method clearly.

(Total for Question 4 is 7 marks)

End of Questions

Gold Mark Scheme

Q1

Question Number	Scheme	Marks
(a)	PQ: $m_1 = \frac{10-2}{9-(-3)}$ $(=\frac{2}{3})$ **and** QR: $m_2 = \frac{10-4}{9-a}$	M1
(b)	$m_1 m_2 = -1$: $\frac{8}{12}\times\frac{6}{9-a} = -1$ $a = 13$ (*)	M1 A1 (3)
Alt for (a)	(a) Alternative method (Pythagoras) Finds **all three** of the following	
	$(9-(-3))^2 + (10-2)^2$, (*i.e.*208), $(9-a)^2 + (10-4)^2$, $(a-(-3))^2 + (4-2)^2$	M1
	Using Pythagoras (correct way around) e.g. $a^2 + 6a + 9 = 240 + a^2 - 18a + 81$ to form equation	M1
	Solve (or verify) for a, $a = 13$ (*)	A1 (3)
	(b) Centre is at (5, 3)	B1
	$(r^2 =)\ (10-3)^2 + (9-5)^2$ or equiv., or $(d^2 =)\ (13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65$ or $x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1 (5)
Alt for (b)	Uses $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$ and substitutes (-3, 2), (9, 10) and (13, 4) then eliminates one unknown	M1
	Eliminates second unknown	M1
	Obtains $g = -5, f = -3, c = -31$ or $a = 5, b = 3, r^2 = 65$	A1, A1, B1cao (5) **[8]**

Notes	
(a)	**M1**-considers gradients of PQ and QR -must be y difference / x difference (or considers three lengths as in alternative method) **M1** Substitutes gradients into product = -1 (or lengths into Pythagoras' Theorem the correct way round) **A1** Obtains $a = 13$ with no errors by solution or verification. Verification can score 3/3.
(b)	Geometrical method: **B1** for coordinates of centre – can be implied by use in part (b) **M1** for attempt to find r^2, d^2, r or d (allow one slip in a bracket). **A1** cao. These two marks may be gained implicitly from circle equation **M1** for $(x \pm 5)^2 + (y \pm 3)^2 = k^2$ or $(x \pm 3)^2 + (y \pm 5)^2 = k^2$ ft their (5,3) Allow k^2 non numerical. **A1** cao for whole equation and rhs must be 65 or $\left(\sqrt{65}\right)^2$, (similarly B1 must be 65 or $\left(\sqrt{65}\right)^2$, in alternative method for (b))

Question Number	Scheme	Marks
Further alternatives	(i) A number of methods find gradient of PQ = 2/3 then give perpendicular gradient is −3/2 This is **M1**	M1
	They then proceed using equations of lines through point Q or by using gradient QR to obtain equation such as $\frac{4-10}{a-9} = -\frac{3}{2}$ **M1** (may still have x in this equation rather than a and there may be a small slip)	M1
	They then complete to give (a)= 13 **A1**	A1
	(ii) A long involved method has been seen finding the coordinates of the centre of the circle first. This can be done by a variety of methods Giving centre as (c, 3) and using an equation such as $(c-9)^2 + 7^2 = (c+3)^2 + 1^2$ (equal radii) or $\frac{3-6}{c-3} = -\frac{3}{2}$ **M1** (perpendicular from centre to chord bisects chord)	M1
	Then using c (= 5) to find a is **M1**	M1
	Finally $a = 13$ **A1**	A1
	(iii) Vector Method: States **PQ**. **QR** = 0, with vectors stated 12**i** +8**j** and $(9 - a)$**i** + 6**j** is **M1**	M1
	Evaluates scalar product so $108 - 12a + 48 = 0$ (**M1**)	M1
	solves to give $a = 13$ (**A1**)	A1

Q2

Question Number	Scheme	Marks
	(a) $(10-2)^2+(7-1)^2$ or $\sqrt{(10-2)^2+(7-1)^2}$	M1 A1
	$(x\pm 2)^2+(y\pm 1)^2=k$ (k a positive <u>value</u>)	M1
	$(x-2)^2+(y-1)^2=100$ (Accept 10^2 for 100)	A1
	(Answer only scores full marks)	(4)
	(b) (Gradient of radius =) $\dfrac{7-1}{10-2}=\dfrac{6}{8}$ (or equiv.) Must be seen in part (b)	B1
	Gradient of tangent $=\dfrac{-4}{3}$ (Using perpendicular gradient method)	M1
	$y-7=m(x-10)$ Eqn., in any form, of a line through (10, 7) with any numerical gradient (except 0 or ∞)	M1
	$y-7=\dfrac{-4}{3}(x-10)$ or equiv (ft gradient of <u>radius</u>, dep. on <u>both</u> M marks)	A1ft
	$\{3y=-4x+61\}$ (N.B. The A1 is only available as <u>ft</u> after B0)	
	The unsimplified version scores the A mark (isw if necessary... subsequent mistakes in simplification are not penalised here.	
	The equation must at some stage be <u>exact</u>, not, e.g. $y=-1.3x+20.3$	(4)
	(c) $\sqrt{r^2-\left(\dfrac{r}{2}\right)^2}$ Condone sign slip if there is evidence of correct use of Pythag.	M1
	$=\sqrt{10^2-5^2}$ or numerically exact equivalent	A1
	$PQ\left(=2\sqrt{75}\right)=10\sqrt{3}$ Simplest surd form $10\sqrt{3}$ required for final mark	A1
		(3) **11**
	(b) 2[nd] M: Using (10, 7) to find the equation, in any form, of a straight line through (10, 7), with any numerical gradient (except 0 or ∞). <u>Alternative:</u> 2[nd] M: Using (10, 7) and an m value in $y=mx+c$ to find a value of c. (b) <u>Alternative</u> for first 2 marks (differentiation): $2(x-2)+2(y-1)\dfrac{dy}{dx}=0$ or equiv. B1 Substitute $x=10$ and $y=7$ to find a value for $\dfrac{dy}{dx}$ M1 (This M mark can be awarded generously, even if the attempted 'differentiation' is not 'implicit'). (c) <u>Alternatives:</u> To score M1, must be a <u>fully</u> correct method to obtain $\dfrac{1}{2}PQ$ or PQ. 1[st] A1: For alternative methods that find PQ directly, this mark is for an <u>exact numerically correct version</u> of PQ.	

Q3

Question	Scheme	Marks	AOs
(a)	Attempts to complete the square $(x\pm3)^2+(y\pm5)^2=...$	M1	1.1b
	(i) Centre $(3,-5)$	A1	1.1b
	(ii) Radius 5	A1	1.1b
		(3)	
(b)	Uses a sketch or otherwise to deduce $k=0$ is a critical value	B1	2.2a
	Substitute $y=kx$ in $x^2+y^2-6x+10y+9=0$	M1	3.1a
	Collects terms to form correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$	A1	1.1b
	Attempts $b^2-4ac...0$ for their a, b and c leading to values for k $"(10k-6)^2-36(1+k^2)...0"\rightarrow k=...,...$ $\left(0 \text{ and } \frac{15}{8}\right)$	M1	1.1b
	Uses $b^2-4ac>0$ and chooses the outside region (see note) for **their** critical values (Both a and b must have been expressions in k)	dM1	3.1a
	Deduces $k<0, k>\frac{15}{8}$ oe	A1	2.2a
		(6)	
			(9 marks)

Notes

(a)

M1: Attempts $(x\pm3)^2+(y\pm5)^2=..$

This mark may be implied by candidates writing down a centre of $(\pm3,\pm5)$ or $r^2=25$

(i) A1: Centre $(3,-5)$

(ii) A1: Radius 5. Do not accept $\sqrt{25}$

Answers only (no working) scores all three marks

(b)

B1: Uses a sketch or their subsequent quadratic to deduce that $k=0$ is a critical value

You may award for the correct $k<0$ but award if $k\leqslant 0$ or even with greater than symbols

M1: Substitutes $y=kx$ in $x^2+y^2-6x+10y+9=0$ or their $(x\pm3)^2+(y\pm5)^2=...$ to form an equation in just x and k. It is possible to substitute $x=\frac{y}{k}$ into their circle equation to form an equation in just y and k

A1: Correct 3TQ $(1+k^2)x^2+(10k-6)x+9=0$ with the terms in x collected. The "= 0" can be implied by subsequent work. This may be awarded from an equation such as $x^2+k^2x^2+(10k-6)x+9=0$ so long as the correct values of a, b and c are used in $b^2-4ac...0$.

FYI The equation in y and k is $(1+k^2)y^2+(10k^2-6k)y+9k^2=0$ oe

M1: Attempts to find two critical values for k using $b^2-4ac...0$ or $b^2...4ac$ where ... could be "=" or any inequality.

dM1: Finds the outside region using their critical values. Allow the boundary to be included. It is dependent upon all previous M marks and both a and b must have been expressions in k.

Note that it is possible that the correct region could be the inside region if the coefficient of k^2 in $4ac$ is larger than the coefficient of k^2 in b^2 Eg

$b^2-4ac=(k-6)^2-4\times(1+k^2)\times9>0\Rightarrow-35k^2-12k>0\Rightarrow k(35k+12)<0$

Q4

Question	Scheme		Marks	AOs
	Finds radius gradient $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)		M1	3.1a
	Finds gradient perpendicular to their radius using $-\frac{1}{m}$		M1	1.1b
	Finds (equation and) y intercept of tangent (see note below)		M1	1.1b
	Obtains a correct value for y intercept of their tangent i.e. 35 or -23		A1	1.1b
	Way 1: Deduces gradient of second tangent	Way 2: Deduces midpoint of PQ from symmetry ((0,6))	M1	1.1b
	Finds (equation and) y intercept of second tangent	Uses this to find other intercept	M1	1.1b
	So obtains distance PQ=35+23=58*		A1*	1.1b
			(7)	
				(7 marks)

Notes

M1: Calculates $\frac{11-6}{10-(-2)}$ or $\frac{1-6}{10-(-2)}$ (m)

M1: Finds $-\frac{1}{m}$ (correct answer is $-\frac{12}{5}$ or $\frac{12}{5}$) This is referred to as m' in the next note.

M1: Attempts $y-11=their\left(-\frac{12}{5}\right)(x-10)$ or $y-1=their\left(\frac{12}{5}\right)(x-10)$ and puts $x=0$, or uses vectors to find intercept e.g. $\frac{y-11}{10}=-m'$

A1: One correct intercept 35 or -23

Qu 17 continued

Way 1:

M1: Uses the negative of their previous tangent gradient or uses a correct $-\frac{12}{5}$ or $\frac{12}{5}$

M1: Attempts the second tangent equation and puts $x = 0$ or uses vectors to find intercept

e.g. $\frac{11-y}{10} = m'$

Way 2:

M1: Finds midpoint of PQ from symmetry. (This is at $(0,6)$)

M1: Uses this midpoint to find second intercept or to find difference between midpoint and first intercept. e.g. $35 - 6 = 29$ then $6 - 29 = -23$ so second intercept is at $(-23, 0)$

Ways 1 and 2:

A1*: Obtain 58 correctly from a valid method.

Platinum Questions

Calculators may not be used

The total mark for this section is 20

1 A point P lies on the curve with equation

$$x^2 + y^2 - 6x + 8y = 24.$$

Find the greatest and least possible values of the length OP, where O is the origin.

(6)

(Total for Question 1 is 6 marks)

2 The line with equation $y = mx$ is a tangent to the circle C_1 with equation

$$(x + 4)^2 + (y - 7)^2 = 13.$$

(a) Show that m satisfies the equation

$$3m^2 + 56m + 36 = 0.$$

(4)

The tangents from the origin O to C_1 touch C_1 at the points A and B.

(b) Find the coordinates of the points A and B.

(8)

Another circle C_2 has equation $x^2 + y^2 = 13$. The tangents from the point $(4, -7)$ to C_2 touch it at the points P and Q.

(c) Find the coordinates of either the point P or the point Q.

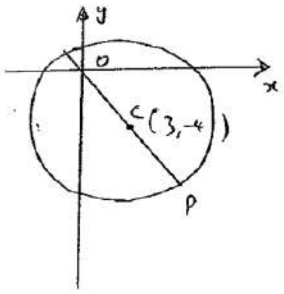
(2)

(Total for Question 2 is 20 marks)

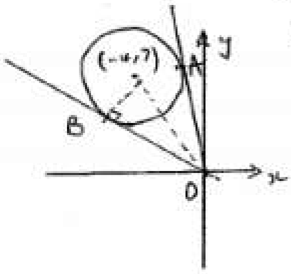
End of Questions

Platinum Mark Scheme

1.

Question Number	Scheme	Marks
1	$(x-3)^2+(y+4)^2 = 24+9+16 = 49$	M1
	Curve is circle, centre $(3, -4)$, radius 7	A1
	 $OC = \sqrt{3^2+4^2} = 5$	M1
	Greatest length $OP = 5 + r$ (or least)	M1
	$= \underline{12}$	A1
	Least length $= r - 5 = \underline{\underline{2}}$	A1
		(6)

2.

Qn 4 (a)	$(x+4)^2+(y-7)^2=13$ and $y=mx$	
	$\therefore (x^2+8x+16)+(m^2x^2-14mx+49)=13$	M1
	$(1+m^2)x^2+(8-14m)x+52=0$ (3 t-quad)	A1
	Touches, so "$b^2=4ac$" $(8-14m)^2=4\times 52\times(1+m^2)$	M1 ($b^2=4ac$
	$(4-7m)^2=52+52m^2$	
	$\therefore$ $3m^2+56m+36=0$ *	A1 cso (4)
(b)	$(3m+2)(m+18)=0$	M1
	$\therefore m=-2/3$ or -18 (both m)	A1
	 Let A or B be (X,Y)	
	then $(X^2+Y^2)+13=4^2+7^2=65$	
	$X^2+Y^2=52$	M1, A1
	$Y=-\frac{2}{3}X \Rightarrow \frac{13}{9}X^2=52 \Rightarrow X=\pm 6$	M1, A1
	From the configuration $X_B=-6 \therefore Y_B=+4$: B is $(-6,4)$	
	$Y=-18X \Rightarrow 325X^2=52 \Rightarrow X^2=\frac{4}{25}$	
	Again $X<0$ for A $\therefore X_A=-\frac{2}{5}$; $Y_A=\frac{36}{5}$	
	$A=(-\frac{2}{5},\frac{36}{5})$	M1, A1 (8)
(c)	Situation is a translation of problem in (b) by $\begin{pmatrix}4\\-7\end{pmatrix}$	M1
	So P, Q are $\begin{pmatrix}-6\\4\end{pmatrix}+\begin{pmatrix}4\\-7\end{pmatrix}$ and $(-\frac{2}{5},\frac{36}{5})+(4,-7)$	
	$=(-2,-3)$ and $(\frac{18}{5},\frac{1}{5})$	A1 (either) (2)